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Application of hybrid AHP-TOPSIS technique in analyzing the braking performance sensitivity of organic-ceramic fibrous reinforced friction composites

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In this research work, brake pad friction composite materials based on binary and ternary combinations of ceramic-organic fibres have been realized, following evaluation of braking performance parameters on Krauss friction testing machine adopting ECE R-90 regulations and PVW-32 standard test protocol. The obtained experimental performance data has been further used for the assessment of performance based ranking using hybrid AHP-TOPSIS technique. The order of relative weights or priority order of performance defining criteria as obtained by AHP is μ-Performance (μ_p) ~ Stability Coefficient (α) > % μ-Recovery > Wear (g) > % μ-Fade > DTR (°C) > Friction Fluctuations (Δμ) > Variability Coefficient (γ). The consistency verification highlights that λ_{max} = 8.0391, consistency index = 0.0045579, and consistency ratio (CR) = 0.00323 << 0.1 (upper bound limit for acceptance of CR). The aramid fibre 5-7.5 wt.% in combination with other fibres 25-22.5 wt.% (for binary combination) and 12.5/12.5 - 11.25/11.25 wt.% (for ternary combinations) has been found to impart the best overall performance level relative to other combos of the friction composites under investigation. The sensitivity analysis of performance defining criteria's and ranking orders of the compositions in the respective formulations gives robust/stable observations as the weights changes from ± (10-20)%. The hybrid AHP-TOPSIS technique in-conjunction-with sensitivity analysis might sever as an effective tool in the decision making whenever there are finite material alternatives and finite performance determining criteria having conflicting nature.

Keywords: Friction materials, Hybrid AHP-TOPSIS, Sensitivity analysis, Brake pad, Braking

1 Introduction

The demand for safe, reliable, and efficient braking systems is continually growing at rapid with the growing need for high-speed, light-weight, and fuel-efficient automobiles. This purely depends upon the closely monitored design and development of its various components for efficient braking performance. Brakes pad friction composite materials are one such component often used to stop or slow down the moving/rotating vehicle wheels/rotor; converting the kinetic energy of rotation into thermal energy via sliding and friction at the interface[1]. Thus, it is expected that the brake friction materials should fulfil the stringent norm (e.g. eco-friendly, higher friction coefficient, negligible fading, faster recovery, better wear resistance, low-sensitivity towards load-speed alterations, least noise and vibration propensity, etc.) at all range of braking variables[2, 3]. Thus selecting an appropriate formulation among several that fulfils a set of performance criteria; indeed becomes multi-norms/criteria/attribute decision making. The literature and material scientist suggest that such material must have an amalgam of multi-ingredients broadly classified under the resin, fibres, fillers, and friction additives. The selection of appropriate ingredients, formulation designing, understanding the braking interface tribology, and developing the formulation based on the experimental results are the real challenge faced by scholars and industries. The contribution of various high-performance fibres and their combinations in enhancing the physical, mechanical, and tribo-performance under dynamic operating environments are very well recognized in the literature [1-26]. Thus analytically the decision making for recommending an appropriate formulation becomes multi-ingredients/ alternative and multi-norms/criteria/attribute kind of decisive problem.

The decision-making techniques are briefly reviewed by Jahan *et al.*[4]. They listed various material selection techniques for systematic screening like cost per unit property method, chart method, knowledge-based systems, neural networks, etc. Also, ranking techniques like TOPSIS, ELECTRE, AHP, SAW, fuzzy MCDM, Goal Programming, PROMETHEF, etc. are briefly discussed. These techniques often find application in evaluating real-time industrial problems and others as documented by several scholars. Ishizaka *et al.*[5] employ Groups Analytic Hierarchy

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Process Ordering Method in the selection of new production facilities. Xuebin[6] uses the Non-dominated Sorting Genetic Algorithm (NSGA-II) technique to find Pareto sets and TOPSIS technique with entropy weights for choosing the best compromising solution in economic and environmental power dispatch problems. Satapathy *et al.*[7] employ balancing and ranking method for the evaluation of performance ranking of friction material. Maleque *et al.*[8] employee unit cost per property method and digital logic techniques for evaluating material performance and their ranking. Maniy *et al.*[9] employ a preference selection index method for the ranking of materials alternatives. Delice *et al.*[10] use heuristic evaluation (HE) and AHP approach for the evaluation of usability problems encountered in websites. Zhu *et al.*[11] employ hybrid AHP-PROMETHEF for evaluating the ranking of different friction composites formulations. Shyur *et al.*[12] employ hybrid ANP-TOPSIS for the Vendor selection process. Similarly, hybrid AHP-TOPSIS techniques often used by scholars to solve several routine decisive problems like customer-driven product design process by Lin *et al.*[13], performance improvement of cold chain by Joshi *et al.*[14], ranking evaluation of different flyash based friction formulations by Satapathy *et al.*[15]. Kranthi *et al.*[16] use neural network techniques to study dry sliding wear response of epoxy composites. Mohanty *et al.*[17] use multi-objective genetic algorithms and Pareto fronts for the optimization of the daily production quantity of iron from rotary kiln. Mohanty *et al.*[18] use a neural network and Genetic Algorithm models to correlate mechanical properties with composition and processing parameters of cold-rolled steel sheets.

Motivated from the above literature, the present work lies in examining the ranking analysis of designed and developed friction formulations using a hybrid AHP-TOPSIS technique followed by sensitivity analysis to validate the robustness of the outcome of the decision-making process.

2 Experimental Details and Methodology

2.1 Fabrication, physical, mechanical and surface characterization of the friction composites

Straight phenolic resin (JA-10; binder), barite (inert filler), graphite (SK-304; S.K. Carbon Limited, India, lubricant) reinforced with aramid/Kevlar pulp (IF-258; Twaron, Teijin-Germany), potassium titanate ceramic whiskers (locally supplied by Jayesh Industries, India) and ceramic fibre (SM-70, Standard grade alumino-silicate fibre; M/s Murugappan-Morgan Ltd., India) amounting to 100% by weight were fabricated as per designed formulations (Table 1a, 1b, 1c). The detailed fabrication procedure and processing conditions are adopted as per standard industrial practice[1-3, 19]. The polished samples were characterized for various physical (like density, void content, ash content), mechanical (like hardness, tensile strength, shear strength, impact strength, flexural strength, compressibility), tribo-properties (friction/braking performance assessment *i.e.* friction-fade, friction-recovery, wear, etc.) and surface morphology (SEM). The results found to be within the Industrial accepted norms for commercial applications[1-3, 19].

Table 1a — Design of the SC-series formulation[1, 19].

Ingredients (wt.%)	Composite Nomenclatures				
	SC-1	SC-2	SC-3	SC-4	SC-5
Phenolic resin	15	15	15	15	15
Barite ($BaSO_4$)	50	50	50	50	50
Graphite	5	5	5	5	5
Alumino-silicate ceramic fibres	30	27.5	25	22.5	20
Aramid fibres	0	2.5	5	7.5	10

*25/5 signifies the proportion of ceramic fibre to aramid fibres in the formulation

Table 1b — Design of the TC-series formulation[2, 19].

Ingredients (wt.%)	Composite Nomenclatures				
	TC-1	TC-2	TC-3	TC-4	TC-5
Phenolic resin	15	15	15	15	15
Barite ($BaSO_4$)	50	50	50	50	50
Graphite	5	5	5	5	5
Potassium titanate whiskers	30	27.5	25	22.5	20
Aramid fibres	0	2.5	5	7.5	10

*25/5 signifies the proportion of ceramic whiskers to aramid fibres in the formulation

Table 1c — Design of the TSC-series formulation[3, 19].

Ingredients (wt.%)	Composite Nomenclatures				
	TSC-1	TSC-2	TSC-3	TSC-4	TSC-5
Phenolic resin	15	15	15	15	15
Barite ($BaSO_4$)	50	50	50	50	50
Graphite	5	5	5	5	5
Potassium titanate whiskers	15	13.75	12.5	11.25	10
Alumino-silicate ceramic fibres	15	13.75	12.5	11.25	10
Aramid fibres	0	2.5	5	7.5	10

*12.5/12.5/5 signifies the proportion of ceramic whiskers to ceramic fibres to aramid fibres in the formulation

2.2 Braking tribological performance evaluation method

The braking performance assessment of friction composites under investigation is evaluated on Krauss type Rubber Wheel Direct Current (RWDC) 100C (450 V/50 Hz) tribo-machine. This machine is fully computerized for feeding the operational inputs and has data acquisition capability[1-3, 19]. The standard regulatory test procedure PVW-3212 (Pulse Velocity Wave) confirming to protocol R-90 of ECE (Economic Commission for Europe) has been adopted for the evaluation of cold friction-fade-recovery characteristics of the investigated friction materials[20]. The procedure comprises of two major parts *viz*. bedding cycle and the actual friction assessment test cycles. The bedding cycle was carried out to ensure more than 80% conformal contact, hence the polished pad surface is allowed to slide against the rotor disc for initial 30 brakings of 10 sec. each under a normal braking pressure/load of 2 MPa and rotor disc speed of 660 rpm, such that the temperature rise of the disc did not exceed 280 °C, in case temperature, exceeds the limit, it was brought to cool down to 100 °C intermittently using an air blower. Thus mechanistically bedding cycle ensures controlled friction-induced thermal history and avoids green fade. Then actual testing cycles begin. The friction assessment test cycles consist of seven cycles/runs *viz.* one cold run, five fade runs, and one recovery run. Each run is of 10 brakings with 10 s as the braking duration making the total number of braking operations in the entire test run seventy. In cold friction run initial temperature was maintained at 45 °C. This follows five runs of fade termed as 1st, 2nd, 3rd, 4th, and 5th fade. Each fade run begins with an initial temperature of 100 °C and it rises uninterruptedly until the run completes. The subsequent fade runs proceed similarly. Finally, the recovery run begins in which the disc was allowed to cool down to a temperature of 100 °C aided by air blower. The friction force and the temperature rise of the disc surface are recorded after every cycle of the braking in a synchronized manner. With the help of in-build software in the machine, averaging and plotting of friction coefficients (μ) were done. For each sample, the experiment was repeated twice and the results obtained are within a 95% confidence level. The wear of the tribo-pairs *i.e.* disc and brake pad are measured in terms of change in the thickness and weight loss before and after the test[1-3, 19].

2.3 Ranking analysis using hybrid AHP-TOPSIS technique

Multi-Criteria-Decision-Making (MCDM) techniques are the prospective mathematical tools often used to aid decision making whenever there are finite alternative and finite conflicting selecting criteria. These are used in-conjunction-with qualitative analysis while taking decisions. These aid in finding the most preferable order of ranking of alternatives based upon the selecting criteria's and almost used across diversified areas of problems like society, economics, military, management, etc.[4-19, 21-26]. Jan *et al.*[4] reviewed such techniques. Among such techniques, the hybrid AHP-TOPSIS technique is often used by decision-makers as it (i) integrates both the quantitative and qualitative aspects of the judgment by-means-of pair-wise relative assessment of selecting criteria using Saaty' scale. The selecting criteria's may be conflicting in nature (ii) aid decision-maker in a better understanding of the problem, thereby judging the best decision suiting the goal. The illustrations of various steps of the algorithm are discussed in section 2.3.1 and section 2.3.2.

2.3.1 Algorithm for determining relative weights of criteria via the AHP method

AHP algorithm is a powerful and flexible technique introduced by Thomas L. Saaty[10-15, 21-26] in the 1970s. Following are the important steps it involves:

Step-1: Construction of the hierarchy chart: This step involves an in-depth understanding of the decision-making problem. The scholar has to make, descriptive discussion or brainstorming sessions with the subject experts, aided by literature knowledge, and establish the goal/objective of the decision-making problem. Thereafter, performance determining criteria's (PDCs), their description, and their implications/nature in-relation-to established goal need to be ascertained (Table 2). This followed by selecting material alternatives over which ranking has to be performed. All the collected information is to be arranged as per the hierarchy chart shown in Fig. 1 for the present investigation.

Step-2: Construction of the pair-wise comparison matrix and evaluation of consistency: In this step, the pair-wise comparison matrix is developed by assigning numerical scores as per Sattay's 1-9 scale (Table 3a), thus incorporating human judgment in the decision-making process. One criterion is pair-wise compared with criteria in the next level (e.g. C_1 & C_2, C_1 & C_3, etc.) and comparison between similar criteria (e.g. C_1 & C_1, C_2 & C_2, etc.) results in the score of unity. In this way, the pair-wise comparison matrix C looks like:

Table 2 — Description of PDCs for the evaluation/ranking of friction composites[1-3, 15, 17, 19].

PDCs No.	Performance Determining Criteria (PDC)	Implications	Brief description of PDCs
PDC-1	Friction-performance (μ_p)	Higher-the-better	The average coefficient-of-friction of all the 70 brakings of the actual test cycle taken after 1 s at a temperature greater than 100 °C.
PDC-2	Stability coefficients (α)	Higher-the-better	It is the ratio of (μ_p/μ_{max}). It represents stability in the frictional response.
PDC-3	Friction recovery performance (% μ-recovery)	Higher-the-better	It is the ratio of ($\mu_{r/}\mu_p$). Once the brake lining cools down (by air blower in this case or with the release of brakes) the revival of braking-efficiency to its original is termed as recovery.
PDC-4	Friction fade performance (% μ-fade)	Lower-the-better	It is the ratio of difference between (μ_p, μ_f) and μ_p. Higher fade signifies poor performance.
PDC-5	Variability coefficients (γ)	Lower-the-better	It is the ratio of (μ_{min}/μ_{max}). It represents variability in the frictional response.
PDC-6	Fluctuations in frictional response ($\Delta\mu= \mu_{max}- \mu_{min}$)	Lower-the-better	It is the difference between maximum and minimum registered friction coefficient during actual cycle tests.
PDC-7	Wear (g)	Lower-the-better	The continuous loss of material from the surface of brake pad due to thermo-mechanical and shear stresses caused by the frictional interactions during braking. Lower wear signifies higher operational life expectancy.
PDC-8	Disc-temperature rise (DTR, °C)	Lower-the-better	The rise in rotor/disc temperature during the actual cycle test as a result of conversion of the kinetic energy of rotating disc into thermal energy because of friction between the interfaces.

Subscript p, r, f, min, max to μ refers to performance, recovery, fade, maximum and minimum coefficient of friction

$$C_{n\times n} = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} 1 & C_{12} & \cdots & C_{1n} \\ C_{21} & 1 & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & 1 \end{bmatrix} \end{array}$$

or [Ci j] where i (row) = 1, 2, …n and j (column) = 1, 2, …n and C_{ij} is the quantified degree of preference of ith criteria (row) over jth criteria (column), hence [Ci j] is nth order square matrix having scores at diagonal equals to 1 and $C_{ij} = \frac{1}{C_{ji}}$.

For n-criteria's, there were $\frac{n(n-1)}{2}$ pair-wise comparisons. For a developed pair-wise comparison matrix C, the relative weight vector w may be ascertained by finding a solution to the characteristic equation $C.w = \lambda_{max}.w$, where λ_{max} is the maximal Eigen value and w refers to the weight vector of the actual absolute weights or Eigen vector associated with the Eigen value[26]. For, perfect consistency $\lambda_{max} = n$ or rank = 1. However, inconsistencies in the priorities determination may lead to dissimilar λ_{max} values, in-such-cases $\lambda_{max} \approx n$. Thereafter, a consistency test evaluation must be performed via the Consistency Index (CI) $= \frac{(\lambda_{max} - n)}{(n-1)}$. For consistency of results $\lambda_{max} \cong n$ or rank = 1 or CI=0. In the end, the extent of consistency or consistency verification needs to be evaluated by computing the Consistency Ratio (CR) = $\frac{CI \text{ (Consistency Index)}}{RI \text{ (Random Index)}}$

For consistency, CR ≤ 0.1 or 10% otherwise pair-wise comparison matrix needs to reconstruct to minimize inconsistency repeatedly. Thus, the CR metric evaluates the consistency of decision-makers. The random index (RI) values shown in Table 3b is the average of the consistency index of 500 randomly generated matrices [10-15, 21-26].

2.3.2 Algorithm for determining the ranking of alternatives via TOPSIS approach

The methodology of TOPSIS was introduced by Hwang and Yoon[11, 13-19, 21-26]. Its effectiveness in

Table 3a — The fundamental relational scale (Sattay's 1-9 scale) for pair-wise comparison[15,18, 19, 21]

Intensity of importance on an absolute scale	Verbal judgment of preferences
1	'A' is equally preferred to 'B'
2	'A' is equal to moderately preferred over 'B'
3	'A' is moderately preferred over 'B'
4	'A' is moderately to strongly preferred over 'B'
5	'A' is strongly preferred over 'B'
6	'A' is strongly to very strongly preferred over 'B'
7	'A' is very strongly preferred over 'B'
8	'A' is very strongly to extremely preferred over 'B'
9	'A' is extremely preferred over 'B'
Reciprocals	If activity 'A' has one of the above number assigned to it when compared with activity 'B', then 'B' has the reciprocal value when compared with 'B'

Table 3b — Random index (RI) for the pair-wise comparison matrix[10, 15, 19].

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

evaluating practicable results while solving the real-life decisive problem of various domains is very well reported in the literature [11-19, 21-26]. Following are the important steps it involves:

Step-1: Construction of decision matrix D: In this step, the data correspond to performance determining criteria's (say *n*-criteria) of each alternative (say *m*-alternatives) are arranged in the form of decision matrix (say matrix D of $m \times n$ order) shown below:

$$D_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix} \end{array}$$

where $C_1, C_2, ..., C_n$ are the *n*-criteria and
$A_1, A_2, ..., A_m$ are the *m*-alternatives

The element p_{ij} is the performance data of the ith alternative (A_i) with-respect-to the jth criteria (C_j) where i =1, 2… *m*, and j = 1, 2… *n*.

Step-2: Development of Normalized matrix: The entries of the above decision matrix are normalized using equation 1 to get normalized matrix R = $\{r_{ij}\}$ (of $m \times n$ order). The normalization facilitates a comparison of all criteria's in dimensionless units for inter-attribute comparisons by transforming data into the range 0 to 1.

$$r_{ij} = \frac{p_{ij}}{\left[\sum_{i=1}^{m} p_{ij}^2\right]^{\frac{1}{2}}} \text{, where j = 1, 2… } n \qquad \text{... 1}$$

Step-3: Development of weighted normalized matrix: The obtained normalized matrix R is then transformed into the weighted normalized decision matrix V= $\{V_{ij}\}$ (using eq. 2).

$$V_{ij} = w_j \times r_{ij} \quad \text{where i=1, 2… } m; \text{ j = 1, 2… } n; \ w_j \geq 0;$$

$$\sum_{j=1}^{n} w_j = 1 \qquad \text{... 2}$$

where, w_j are the relative weights as determined by AHP method.

Step-4: Evaluation of positive ideal solution (A^+) and the negative ideal solution (A^-): The weighted normalized matrix is used to determine the positive ideal solution (A^+) and the negative ideal solution (A^-) as per below criteria:

$$A^+ = (v_1^+, v_2^+, ..., v_J^+)$$

$$A^- = (v_1^-, v_2^-, ..., v_J^-)$$

whereas,

$$v_j^+ = \begin{cases} \max V_{ij}, \text{ if j is a benefit criteria or larger-the-better} \\ \min V_{ij}, \text{ if j is a cost criteria or smaller-the-better} \end{cases}$$

whereas j = 1,2, …, *n*

$$v_j^- = \begin{cases} \max V_{ij} \text{ , if j is a benefit criteria or larger-the-better} \\ \min V_{ij} \text{ , if j is a cost criteria or smaller-the-better} \end{cases}$$

Step-5: Computation of Euclidian distance (D): This step involves the computation of Euclidean distance between positive ideal solution and the negative ideal solution for each alternative, using eq.3,

Table 4 — Pair-wise comparison matrix of PDCs and their relative weights.

	PDC-1	PDC-2	PDC-3	PDC-4	PDC-5	PDC-6	PDC-7	PDC-8	Relative Weights (W_j)
PDC-1	1	1	2	3	5	4	2	3	0.24082
PDC-2	1	1	2	3	5	4	2	3	0.24082
PDC-3	½	1/2	1	2	3	2	1	2	0.13262
PDC-4	1/3	1/3	1/2	1	3/2	3/2	1/2	1	0.07505
PDC-5	1/5	1/5	1/3	2/3	1	1	2/5	2/3	0.04977
PDC-6	¼	1/4	1/2	2/3	1	1	1/2	1	0.06010
PDC-7	½	1/2	1	2	5/2	2	1	2	0.12952
PDC-8	1/3	1/3	1/2	1	3/2	1	1/2	1	0.07131
								Total =	1.00000

Table 5a — Experimental data of SC-series friction materials[1, 19].

SC Composites	PDC-1 (μ_p)	PDC-2 (Stability Coefficient)	PDC-3 (%μ-recovery)	PDC-4 (%μ-fade)	PDC-5 (Variability Coefficient)	PDC-6 (Friction Fluctuations)	PDC-7 (Wear)	PDC-8 (DTR)
SC-1	0.363	0.672	138.567	11.019	0.150	0.459	6.900	538
SC-2	0.378	0.712	140.476	6.878	0.092	0.482	2.900	525
SC-3	0.354	0.753	132.768	27.119	0.100	0.423	2.900	475
SC-4	0.315	0.847	111.111	54.603	0.183	0.304	2.050	470
SC-5	0.217	0.604	163.594	64.516	0.214	0.282	0.600	300

$$D_i^+ = \sqrt{\sum_{j=1}^{n}(v_j^+ - v_{ij})^2} \quad \text{where i = 1, 2, ..., } m$$

$$D_i^- = \sqrt{\sum_{j=1}^{n}(v_j^- - v_{ij})^2} \qquad \ldots 3$$

Step-6: Computation of Closeness Coefficient (CC): This step involves the computation of relative closeness or the overall preference or Closeness Coefficient (CC) to the ideal solution for each alternatives using eq. 4. As D_i^+ *and* D_i^- both > 0, hence CC $\in$ (0, 1).

$$CC_i = \frac{D_i^-}{D_i^+ + D_i^-} \qquad for\ i = 1,\ 2,\ ....,m \qquad ...\ 4$$

Step-7: Ranking determination: In this step, the alternatives ranking order is determined as per their evaluated Closeness Coefficient (CC). Larger the magnitude of CC better would be the alternative relative to others. Hence, the alternatives are arranged as per their descending CC values. Accordingly, justification and recommendations are made.

2.3.3 Sensitivity analysis of Criteria's weights

Through sensitivity analysis analysts could investigate the robustness of the outcome of a decision making mathematical model, qualitatively or quantitatively, if the decision making criteria differ from previously assumed under a given set of scenarios. Such analysis enables analysts to make more credible, understandable, compelling, or persuasive recommendations. In the present investigation, sensitivity analysis is carried out by varying (increasing/decreasing) the weights of PDCs ± 10%, ± 20%, and adjusting the weights of other PDCs proportionally such that the sum of weights remains unity[15, 26].

Table 5b — Closeness coefficient and ranking of SC-series friction materials.

SC Composites	CC	Ranking
SC-1	0.3931	5
SC-2	0.6120	3
SC-3	0.6770	1
SC-4	0.6504	2
SC-5	0.5573	4

3 Results and Discussion

The analysis of performance data as per the algorithm is shown in Table 4-7. Table 4 shows the pair-wise comparison matrix and the relative weights. Table 5a lists various PDCs data that corresponds to SC-series as obtained from Krauss friction testing machine and Table 5b presents Closeness Coefficient and ranking of SC-series friction material. Similarly, Table 6 (a-b) & 7 (a-b) represents analysis with-respect-to TC-series and TSC-series formulation.

3.1 Computation of relative weights or priority order of PDCs using AHP

The relative weights or priority order of performance determining criteria are computed as per

Table 6a — Experimental data of TC-series friction materials[2, 19].

TC Composites	PDC-1 (μ_p)	PDC-2 (Stability Coefficient)	PDC-3 (%μ-recovery)	PDC-4 (%μ-fade)	PDC-5 (Variability Coefficient)	PDC-6 (Friction Fluctuations)	PDC-7 (Wear)	PDC-8 (DTR)
TC-1	0.341	0.680	110.850	0.293	0.230	0.383	2.820	498
TC-2	0.338	0.690	112.130	0.592	0.240	0.374	2.700	483
TC-3	0.366	0.780	120.220	4.372	0.270	0.343	2.650	456
TC-4	0.359	0.760	118.660	41.783	0.140	0.407	2.600	480
TC-5	0.364	0.750	124.180	44.505	0.180	0.400	2.250	453

the AHP algorithm discussed in section 2.3.1 and shown in Table 4. From the last column of the table, the order of relative weights or priority order of PDCs is μ-Performance (μ_p) [0.24082] ~ Stability Coefficient (α) [0.24082] > % μ-Recovery [0.13262] >Wear (g) [0.12952] > % μ-Fade [0.07505] > DTR (°C) [0.07131] > Friction Fluctuations (Δμ) [0.06010] > Variability Coefficient (γ) [0.04977]. The consistency verification highlights that λ_{max} = 8.0391, Consistency Index = 0.0045579, RI = 1.41 (against 8-criteria) and Consistency Ratio (CR) = 0.00323 << 0.1 (upper bound limit for acceptance of CR). Henceforth, the relative weights are consistent and could further be used as input to the TOPSIS algorithm.

3.2 Ranking of friction formulations using TOPSIS and sensitivity analysis

The friction composite materials formulations under investigation are systematically shown in Table 1 (a-c). It comprises straight phenolic resin, barite filler, and graphite lubricant to forms master-batch of constant weight proportion reinforced by a complementary combination of fibres as per designed proportions. This aid in understanding the role of fibrous combination towards physical, mechanical, thermal, and tribology of braking performance under serve experimental setup/simulations. Therefore, there is a binary formulation of organic-ceramic fibres *i.e.* aramid/ alumino-silicate ceramic fibres (SC-series) and organic-ceramic whiskers *i.e.* aramid/potassium titanate ceramic whiskers (TC-series). Similarly, there is a ternary formulation of organic-ceramic-whiskers fibres *i.e.* aramid-potassium titanate ceramic whiskers-alumino-silicate ceramic fibres (TSC-series), where an equal proportion of ceramic ingredients is maintained. Here, motto is to determine the ranking order of the composites in respective formulations and discussing how the selection of fibrous combination and their weight proportions affects ranking order. This exercise demonstrates that careful fibrous combination selection and their weight fraction aid formulation designer in adjusting/replacing one fibre in-conjunction with other

Table 6b — Closeness coefficient and ranking of TC-series friction material.

TC Composites	CC	Ranking
TC-1	0.2804	5
TC-2	0.6929	3
TC-3	0.7571	1
TC-4	0.7078	2
TC-5	0.2835	4

keeping the same functional performance level as determined experimentally for cost optimization and development for commercial application.

The analysis shown in Table 5b, Table 6b and Table 7b shows interesting observations (i) SC-3 > SC-4 > SC-2 > SC-5 > SC-1 binary ceramic/organic fibre combination having 25/5 proportion *i.e.* SC-3 composites shows the highest ranking, while combinations having 22.5/7.5 and 27.5/2.5 proportions *i.e.* SC-4/SC-2 composites shows next lower level ranking, whereas combination having 20/10 proportion *i.e.* SC-5 composites shows the next lower level ranking and the combination having 30/0 proportion *i.e.* SC-1 composites shows the lowest ranking order (ii) TC-3 > TC-4 > TC-2 > TC-5 > TC-1 binary inorganic-whiskers/organic fibre combination having 25/5 proportion *i.e.* TC-3 composites shows the highest ranking, while combinations having 22.5/7.5 and 27.5/2.5 proportions *i.e.* TC-4/TC-2 composites shows next lower level ranking, whereas combination having 20/10 proportion *i.e.* TC-5 composites shows the next lower level ranking and combination having 30/0 proportion *i.e.* TC-1composites shows the lowest ranking order (iii) TSC-3 > TSC-4 > TSC-2 > TSC-5 > TSC-1 ternary inorganic-whiskers/ceramic/organic fibre combination having 12.5/12.5/5 proportion *i.e.* TSC-3 composites shows the highest ranking, while combinations having 11.25/11.25/7.5 and 13.75/13.75/2.5 proportions *i.e.* TSC-4/TSC-2 composites shows next level lower ranking, whereas combination having 10/10/10 proportion *i.e.* TSC-5 composites shows the next lower ranking and combination having 15/15/0 proportion *i.e.* TSC-1composites shows the lowest

Table 7a — Experimental data of TSC-series friction materials[3, 19].

TSC Composites	PDA-1 (μ_p)	PDA-2 (Stability Coefficient)	PDA-3 (%μ-recovery)	PDA-4 (%μ-fade)	PDA-5 (Variability Coefficient)	PDA-6 (Friction Fluctuations)	PDA-7 (Wear)	PDA-8 (DTR)
TSC-1	0.386	0.670	113.470	13.731	0.100	0.519	4.350	525
TSC-2	0.351	0.660	113.390	38.746	0.110	0.473	1.600	448
TSC-3	0.377	0.710	115.920	58.355	0.130	0.463	1.750	455
TSC-4	0.321	0.660	136.140	74.143	0.130	0.419	1.600	400
TSC-5	0.346	0.660	131.790	75.434	0.150	0.444	1.450	425

ranking order. Comprehensively, it could be inferred that friction composite compositions having 5 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows the highest order, while compositions having 7.5 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows next lower order, while compositions having 2.5 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows next lower order, whereas compositions having 10 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows next lower order and compositions having 0 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows lowest ranking order. Thus, aramid fibre 5-7.5 wt.% is considered optimal in combination with other fibres 25/12.5 to 25/11.25 wt.% that imparts the highest performance level to the friction composites. The other combinations impart a lower performance level. The contribution of aramid fibre in enhancing pre-form strength during fabrication and significant tribological role while braking instances is very well documented in the literature[1-3, 7, 11, 19]. This fact is further validated/ proved during difficulties in the pre-form fabrication of such compositions. This highlights the significant role of the binary/ternary combination of high-performance fibrous ingredients in monitoring the overall braking performance of brake friction composite material.

The sensitivity analysis of PDCs and ranking orders of the compositions in their respective formulations computationally gives robust/stable observations. The order of ranking remains insensitive/robust/stable as the weights of PDCs changes from ± (10-20)%. Henceforth, the hybrid AHP-TOPSIS technique in-conjunction-with sensitivity analysis would sever as an effective tool in decision making whenever there are finite material alternatives and finite performance determining criteria having conflicting nature.

4 Conclusions

The salient outcome from the analysis of braking performance data via hybrid AHP-TOPSIS technique are:

Table 7b — Closeness coefficient and ranking of TSC-series friction material.

TSC Composites	CC	Ranking
TSC-1	0.3012	5
TSC-2	0.7094	3
TSC-3	0.7978	1
TSC-4	0.7461	2
TSC-5	0.6879	4

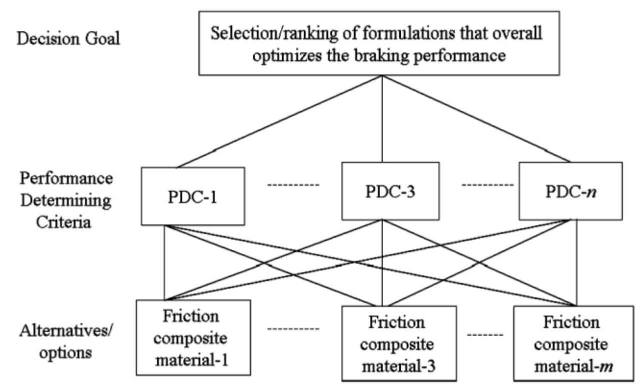

Fig. 1 — The hierarchy chart of the investigated problem using AHP.***

(i) The priority order of PDCs as obtained by AHP analysis are μ-Performance (μ_p) [0.24082] ~ Stability Coefficient (α) [0.24082] > % μ-Recovery [0.13262] > Wear (g) [0.12952] > % μ-Fade [0.07505] > DTR (°C) [0.07131] > Friction Fluctuations (Δμ) [0.06010] > Variability Coefficient (γ) [0.04977].

(vii) The friction composite compositions having 5 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows the highest order, while compositions having 7.5 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows next lower order, while compositions having 2.5 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows next lower order, whereas compositions having 10 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows next lower order and compositions having 0 wt.% aramid fibre in combination with either ceramic or inorganic-whiskers or both shows lowest ranking order.

(iii) The aramid fibre 5-7.5 wt.% in combination with other fibres 25-22.5 wt.% (for binary combination) and 12.5/12.5 - 11.25/11.25 wt.% (for ternary combinations) found to impart the best overall performance level relative to other combos of the friction composites under investigation.

(iv) The sensitivity analysis of performance defining criteria's and ranking orders of the compositions in the respective formulations gives robust/stable observations as the weights changes from ± (10-20)%.

(v) The hybrid AHP-TOPSIS technique in-conjunction-with sensitivity analysis might sever as an effective tool in the decision making whenever there are finite material alternatives and finite performance determining criteria having conflicting nature.

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Abbreviations: MCDM (Multi-Criteria-Decision-Making), AHP (Analytic Hierarchy Process), TOPSIS (Technique for Order Preferences by Similarity to Ideal Solution), PDC (Performance Determining Criteria), PIS (Positive Ideal Solution), NIS (Negative Ideal Solution).

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